

The demand for tickets and travel cards of railway travelers

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Daniël van Vuuren Piet Rietveld

Vrije Universiteit, Amsterdam

Abstract

Train passengers can usually choose between buying a ticket for each trip separately and buying a season ticket. Many railway companies also offer reduced-fare passes. The choice of a train passenger for one of these options depends of course on the fixed and variable charges involved, and on his (quantitative) demand for travel. The analysis of the simultaneous choice of a train passenger for one of the various ticket options and the implications for the number of train kilometres he travels has not received much attention in the literature thus far. The aim of our paper is to discuss various econometric approaches to deal with this problem, and to apply these for the Netherlands. This research is not only relevant for railway companies aiming at maximization of their net returns through an optimal mix of ticket types, but also for governments aiming at stimulation of public transport within certain budgetary limits.

1 Introduction

Public transport companies usually offer their clients various ways to pay for their services. Tickets (single or return) have to be paid every time a trip is made. At the opposite side is an annual card that allows travelers to travel without any limitation during a certain period. In between there may be various cards allowing passengers to travel at reduced fares or, under certain conditions, without further payment. A consequence of such a fare system is that the use of a uniform price p for public transport per km as often used in transport modeling is rather remote from reality. Even if prices of individual tickets are considered we can observe that there is not a constant price per km. Fare systems usually entail a fixed element in the fares and often also a decreasing marginal price per km as the distance travelled increases.

Public transport companies have various reasons for such price setting strategies. These relate partly to cost aspects, and partly to monopolistic pricing. At the cost side there is obviously a fixed element in the costs of serving a passenger that is not dependent on the length of the trip. Examples are: costs related to selling at the ticket window, checking the ticket in the train or at the place of entry at the metro station, and costs related to total travel time in the case of services where a bus has to stop to let passengers enter or exit (see Mohring [12]). Thus, as far as fares are related to costs of providing the services one may expect that a fixed element is included in the price of tickets, and that it is attractive

to reduce transaction costs by introducing public transport passes.

The above cost related argument holds in case of price setting in a competitive market. However, even in the absence of fixed elements in the costs of producing transport services, one may still have a nonconstant price per km when transport firms would display monopolistic price setting tendencies. The reason is that monopolistic suppliers will find that price discrimination is profitable because it absorbs parts of the consumer surplus (e.g. see Gravelle and Rees [7]).

In the present paper we will discuss the use of travel cards from the viewpoint of the consumer. Our aim is to develop a method to model the choice of travel cards by travelers. This means that we will not analyse the decision of the transport companies on the appropriate mixture of cards. But of course, once the demand for cards has been modeled this would provide a powerful tool for transport companies to analyse the consequences of changes in the card system including shifts from one type of card to the other, the entry of new customers, or loss of existing customers.

The analysis of the choice of travel cards has some similarity with other choice situations where consumers face varying marginal prices depending on the volume of consumption. For example, in labor markets the system of various levels of marginal taxation leads to rather specific results in terms of the number of hours people wish to work (they may tend to a number of hours per week near to the boundary between a low tax and a high tax tariff). There is also some similarity to block rate pricing sometimes used by suppliers of public utilities such as electricity and water. The kinked budget curve may lead to non-unique optima where consumers are indifferent between two combinations of bundles of goods. A final example which is rather near to the case of travel cards concerns the demand for car kilometres. This can be conceived of as a joint decision of car ownership and car use. Here again the situation may arise that people are indifferent between the alternative of not owning a car but renting one from time to time and owning one combined with a more intensive use.

This paper is organized as follows: In section 2 we provide a description of the current fare system for railway use as it is in the Netherlands, and we also discuss the recent developments of this fare system. In section 3 we resume the most important results from microeconomic theory for our topic. In section 4 an overview of the microeconometrics of discrete/continuous goods is given, after which we turn to our application of train tickets/train kms in section 5. In section 7 we report some first estimates of two simple models that have been estimated with the data that is briefly described in section 6. Section 8 concludes.

2 The fare system for railway use in the Netherlands

Railway use has undergone rather turbulent changes in the Netherlands during the last decade (see Table 1). The total number of km travelled by train increased sharply after 1990 when all students were given a free public transport pass (combined with a forced reduction in their scholarship). This unprecedented increase put substantial pressure on the quality and reliability of the service. Price increases for the standard travelers led to a certain decrease in the total number of travelers around 1994. The volume of travelers with large contracts decreased after 1994 due to changes in the arrangements, implying that it could no longer be used during all days in the week. After 1994 a period of rather modest changes in train tariffs took place. A rapid growth in the revealed demand of standard travellers can be observed

year	traveler kms large contracts	traveler kms standard travelers	total kms
1990	2200	8860	11060
1992	5792	9188	14980
1994	5551	8888	14439
1996	4321	9810	14131
1998	4115	10764	14879

Table 1: Development of passenger kms traveled by train in The Netherlands (*source: NSR*)

type of card	share in passenger km (%)	share in returns (%)	return per passenger km (cents/km)
first class	7	10	23.5
second class	93	90	15.4
full fare	19	27	22.5
single	6	11	26.6
return	9	12	23.3
other	4	5	19.3
reduced fare	22	20	14.0
other tickets (children)	4	1	5.0
railway pass	15	16	17.4
student card	30	22	11.7
international	4	5	19.7

Table 2: Cards of NSR and their contribution to total returns (1996/97) (*source: NSR/MuConsult*)

of about 5% per year; this can partly be explained by a shift from former users of student cards. The development since 1996 indicates, however, that also without such a shift a substantial increase in railway use could be maintained.

The table shows the importance of the large contracts (mainly students) for the Netherlands railways (NSR) in terms of kilometers traveled (39% in 1992, 28% in 1998). In financial terms this category is less important, however, since the receipts per km are rather low. The large contract takes place between the Ministry of Education and NSR. The ministry pays NSR a certain amount of money and reduces the scholarships of students with a larger amount. The students are not free to choose whether or not they want to accept the free public transport pass. For NSR the contract has been interesting given the low marginal cost of their production process. As long as their carriages are not full the costs of serving additional passengers are modest (Rietveld and Roson [15]).

Table 2 gives more information on the various types of cards and their contribution to the total returns of NSR. When we consider second class passengers we find that the largest share in passenger km is for the student cards (30%). It is followed by holders of cards for reduced fares (22%); these cards allow one to travel with a reduction in the fare of 40% or more after the morning peak. Full fare is only paid for

19% of the passenger km. Railway pass holders account for 15% of total distance travelled.

The shares in the total returns are rather different: The largest share (27%) comes from full fare passengers. It is followed by student card holders (22%), reduced fare passengers (20%) and railway pass holders (16%). These passes, once they have been bought give free use of the whole network or a certain trajectory.

The average receipts per km are high for full fare tickets (single and return), they are particularly low for the 'other tickets' category (children), and also for the student card holders. The variance in the receipts per traveller km indicates that the Netherlands railways has been able to arrive at a substantial price discrimination among its customers. Note that the average receipts may be very different from the marginal prices paid by the travellers in some cases. For example, in the case of the railway pass the marginal receipts of an extra passenger km are zero. With the reduced fare card the marginal receipts will also be substantially below the average receipts since the latter include the price of the card giving the right to buy tickets at a reduced fare.

In the rest of this paper we discuss the issue of choice of travel card by railway travelers. Students will be left out of consideration because their ownership of the student card is not a matter of their own choice, but follows from the arrangement between the Ministry of Education and NSR. For the other travelers roughly three main options can be distinguished: full fare tickets, reduction cards giving the right to travel at reduced fares after the morning peak, and a railway pass implying free transport during a longer period once the pass has been bought.

3 The microeconomics of discrete/continuous goods

3.1 The microeconomic framework

Microeconomic textbooks typically assume that prices are linear in quantities. In general it is also assumed that consumer preferences can be represented by a utility function, which we will denote by u . This utility function typically satisfies the following properties for distinct bundles of goods x^1 and x^2 :

- $u(x^1) = u(x^2)$ if and only if $x^1 \sim x^2$;
- $u(x^1) > u(x^2)$ if and only if $x^1 \succ x^2$.

The symbols ' \sim ' and ' \succ ' indicate 'is indifferent to' and 'is preferred to' respectively. Assume now that the individual's consumption space is spanned by two goods, e.g. the *consumption* of train kilometers x_t , and the consumption of all other economic goods x_c . Given some specification of a utility function and given that the price of a train kilometer is fixed, the consumer maximization problem is given by

$$\max_{x_t, x_c} u(x_t, x_c), \quad (1)$$

subject to the budget restriction

$$px_t + x_c \leq y. \quad (2)$$

Here p denotes the price of one train kilometer and y denotes consumer income. Note that the price of the composite good is normalized to 1, which means that x_c can be regarded as a kind of 'net income', that

is income minus expenses on traveling by train. The demand equations $x_t^* \equiv x_t^*(p, y)$ and $x_c^* \equiv x_c^*(p, y)$ follow from the first order condition

$$\frac{u_t(x_t^*, x_c^*)}{u_c(x_t^*, x_c^*)} = p, \quad (3)$$

together with the budget restriction

$$px_t^* + x_c^* = y. \quad (4)$$

With u_t and u_c we denote the derivatives of u w.r.t. x_t and x_c respectively. Note that the inequality sign in (2) is replaced by an equality sign. We allow ourselves to do so by assuming that utility is *nondecreasing* in both commodities. Another assumption that we make throughout this paper is that the utility function is *continuous* and *quasiconcave*. This assumption will not be sufficient for a unique demand equation (as will be shown within a few lines) but is still necessary to do any sort of analysis within our framework.

As many authors have noticed, in daily practice many goods do not have a constant price. As examples one can think of two part tariffs, quantity rebates, etc. The introduction of nonconstant or *nonlinear* prices leads to a budget set that does not have a linear frontier. Therefore, a budget set that is subject to nonlinear prices is often called a ‘nonlinear budget set’. In Figure 1 we give an example of a nonlinear budget set. It is seen that the budget set in this example is *nonconvex*, as price is decreasing with quantity. If price were nondecreasing with quantity, then the budget set would have been *convex*. The grey smooth curve is an indifference curve of a certain consumer, containing all consumption bundles with some given level of utility. The figure clearly displays a well known property of nonconvex budget sets, namely (general) nonuniqueness of the demand function. Another property of the demand function for this kind of budget sets is *discontinuity*, which means that a small change in the price structure can lead to a discontinuous “jump” in demand. (See e.g. Moffitt [11] for an example.)

For nonlinear prices the optimization problem that was given in (1) and (2) transforms into

$$\max_{x_t, x_c} u(x_t, x_c), \quad (5)$$

subject to the budget restriction

$$p(x_t) + x_c \leq y, \quad (6)$$

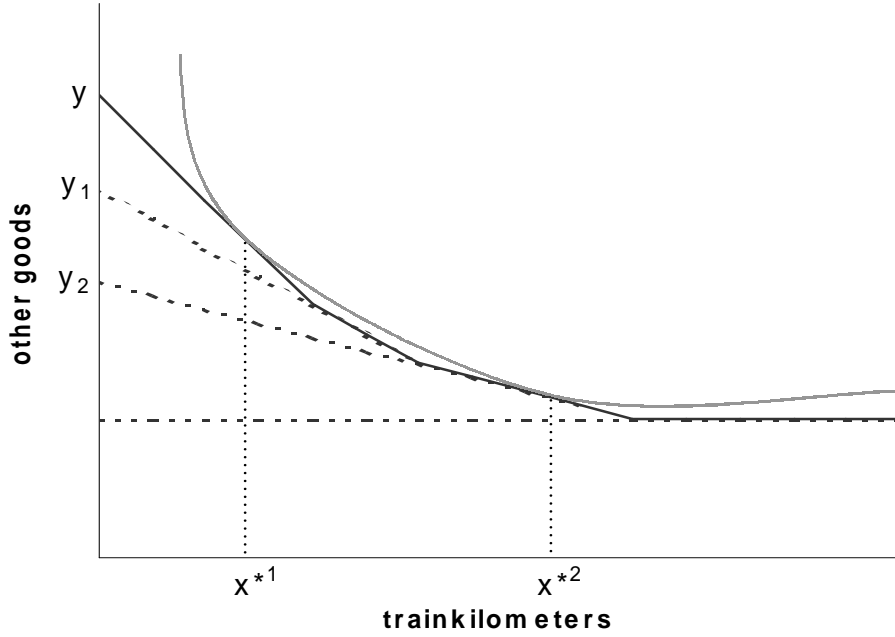
where $p(\cdot)$ is the price function that returns the total price that has to be paid for a given amount of train kilometers.

Most (if not all) empirical studies have dealt with *piecewise linear* price functions. Pudney [14] describes the procedure to determine the demand equations for this case. Let the price function be given by

$$p(x_t) = \sum_{i=1}^k (\alpha_i + \beta_i x_t) 1(x_t \in (x_t^{i-1}, x_t^i]),$$

where “1” is the *indicator function*, k denotes the number of intervals, and x_t^i ($i = 1, \dots, k$) denote the interval borders, $x_t^k \equiv \infty$. Moreover, $p(0) \equiv 0$. The procedure now proceeds by determining k demand

Figure 1: A nonconvex budget set: Nonuniqueness of Marshallian demand



functions, one for each segment. The demand functions for segment i , x_t^{*i} and x_c^{*i} , follow from the optimization problem

$$\max_{x_t, x_c} u(x_t, x_c),$$

subject to the budget restriction

$$\alpha_i + \beta_i x_t + x_c \leq y.$$

The first order condition in (3) remains the same, with p replaced by β_i . Optimal demand now equals

$$(x_t^*, x_c^*) = \operatorname{argmax}_{(x_t, x_c) \in \{(0, y), (x_t^{*1}, x_c^{*1}), \dots, (x_t^{*k}, x_c^{*k})\}} u(x_t, x_c).$$

Resuming, this procedure consists of the following steps: (i) the budget frontier is cut into disjoint linear pieces, (ii) for each part of the budget frontier the *conditional* demand functions are determined, (iii) the conditional demand function that generates the highest level of utility is chosen as the consumer's demand function.

3.2 Duality

While analyzing the demand for a certain economic good two approaches are possible. The *direct approach*, as Burtless and Hausman [5] call it, specifies a form of the direct or the indirect utility function

and hence derives the consumer demand equations. This is the approach that was outlined in the previous subsection. The second approach arises from the theory of duality, in particular from Roy's theorem. The starting point is to specify the demand function, and hence make use of this theorem in order to derive the indirect utility function. Sometimes it is even possible to derive the direct utility function, but this is not very common. In this context it is important to make notice of the so-called *integrability problem*, which means that the Slutsky matrix must satisfy certain restrictions in order to stay consistent with the theory of utility maximization. However, in the two-goods case this problem does not appear. As an illustration of the *indirect* approach we provide an example of a model for train ticket demand.

Example: A simple model for train ticket demand. The constant elasticity model for train ticket demand may be specified by

$$d = kp^\alpha y^\beta. \quad (7)$$

Here p denotes price, y denotes income, and d denotes the Marshallian demand function at given level of income y , price p and parameter values k , α and β . Burtless and Hausman [5] have applied this same type of model for estimating labor supply functions¹. Denoting the indirect utility function by $v(\cdot)$, the following equation is obtained as a direct application of Roy's identity:

$$kp^\alpha y^\beta = \frac{\partial v(p, y)}{\partial p} \bigg/ \frac{\partial v(p, y)}{\partial y}.$$

Making use of the *implicit function theorem* the following solution is obtained:

$$v(p, y) = k \frac{p^{1+\alpha}}{1+\alpha} + \frac{y^{1-\beta}}{1-\beta}.$$

As one is mostly interested in the demand equation it is most natural to specify this equation first and derive the indirect utility function from it, rather than vice versa. A great advantage of this method over the direct method is that no specification of the (direct) utility function is necessary, which allows for more flexibility of the demand equation.

3.3 Comparative statics

Blomquist [2] gives an extension of comparative statics results from 'classic' utility models to utility models with nonlinear budget constraints. Denote by $d = d(p, y)$ the k -vector of Marshallian demand functions, by p the price vector and by y income. In the case of linear budget constraints the marginal prices are constant and hence the vector p is of dimension k . Let S denote the Slutsky matrix with entries $S_{ij} := \frac{\partial h_i(p, y)}{\partial p_j}$, where $h = h(p, y)$ is the k -vector of Hicksian demand functions. As is known from microeconomic textbooks, the following properties hold in the case of a *linear budget constraint*:

1. The demand functions are homogeneous of degree zero in prices and income: $d(p, y) = d(\lambda p, \lambda y)$ for any $\lambda \geq 0$,
2. Engel aggregation: $p' \frac{\partial d}{\partial y} = 1$,

¹Here price was replaced by wage rate and y was interpreted as 'nonlabor income'. The nonlinearity of the wage rate was due to the nonconstant marginal tax rate.

3. Cournot aggregation: $p' \frac{\partial d}{\partial p} = d$,
4. Slutsky decomposition: $S = \frac{\partial d}{\partial p} + \frac{\partial d}{\partial m} d'$,
5. Linear dependence of substitution effects: $p' S = 0$,
6. The Slutsky matrix is symmetric and negative semidefinite.

Denote now the *nonlinear budget constraint* by $g(x, p) \leq y$ for k -dimensional consumption packages x . Assuming that

- the function $g(x, p)$ is twice continuously differentiable in both arguments,
- there exists a unique differentiable solution to the individual's utility maximization problem (called his Marshallian demand function), and that
- there exists a unique differentiable solution to the individual's dual optimization problem (called his Hicksian demand function),

Denote $g_d := \left. \frac{\partial g}{\partial x} \right|_{x=d}$. Blomquist proves the following properties for the case of nonlinear budget constraints:

1. The demand functions are homogeneous of degree zero in prices and income if and only if $g(x, p)$ is homogeneous of degree one in prices and income: $d(p, y) = d(\lambda p, \lambda y)$ for any $\lambda \geq 0 \iff g(x, p) = \frac{1}{\lambda} g(x, \lambda p)$ for any $\lambda \geq 0$,
2. Engel aggregation: $g'_d \frac{\partial d}{\partial y} = 1$,
3. Cournot aggregation: $g'_d \frac{\partial d}{\partial p} = -\frac{\partial g}{\partial p}$,
4. Slutsky decomposition: $S = \frac{\partial d}{\partial p} + \frac{\partial d}{\partial y} \left(\frac{\partial g}{\partial p} \right)'$,
5. Linear dependence of substitution effects: $g'_d S = 0$,
6. The Slutsky matrix is in general neither symmetric nor negative semidefinite. Since the price vector is of another dimension as k (= the number of goods) most of the times, the Slutsky matrix is even not square.

For the two good case it is possible to derive some very useful formulas, as we will show next. In setting up explicit formulas for the income effects, Blomquist makes use of an important concept while dealing with nonlinear budget sets: the virtual income (\tilde{y}). It is possible to linearize the budget constraints into one linear one with intercept \tilde{y} and slope equal to the slope of the budget frontier in the point that corresponds with actual demand d . It is directly seen that - making use of the quasi-concavity of the utility function - the optimal solution of the 'linearized' problem is equal to the optimal solution of the original problem. Making use of this property, it is possible to express the comparative statics in terms of the comparative statics of the linearized problem:

$$\frac{\partial d}{\partial y} = \frac{\partial d^L}{\partial p} \frac{\partial p}{\partial y} + \frac{\partial d^L}{\partial \tilde{y}} \frac{\partial \tilde{y}}{\partial y}.$$

Here d^L denotes the demand function of the linearized case. Another way to write this formula down is

$$\frac{\partial d}{\partial y} = F \frac{\partial d^L}{\partial y}.$$

The matrix

$$F := \left(I - S \frac{\partial g^2}{\partial^2 x} \Big|_{x=d} \right)^{-1} \quad (8)$$

is called the *fundamental matrix of nonlinearity*, as it transforms the linear income effects into nonlinear ones. We also find this matrix in an expression for the own substitution effect:

$$\frac{\partial d}{\partial p_j} = F \left[S \frac{\partial g^2}{\partial p_j \partial x} - \frac{\partial g}{\partial p_j} \frac{\partial d^L}{\partial \tilde{y}} \right].$$

4 The microeconometrics of discrete/continuous goods

4.1 Stochastic specification

According to Hausman [9] one is able to distinguish between different sources of stochastic disturbances in the case of nonlinear budget constraints. He distinguishes between the following sources of randomness:

1. optimization error by the consumer,
2. measurement error by the data collector,
3. taste variation of the consumer,
4. specification error by the econometrician.

Hausman makes clear that it is possible to incorporate optimization error and measurement error into the model *separately*, and estimate the parameters of the relevant distributions. Note that in a common linear regression model - e.g. in a model with linear prices - this would not be possible, whereas the parameters would not be identified. He also shows how to take account of variation in taste by consumers, but notes that the distributional specification of the varying parameter(s) is a problem. The problem lies in the fact that if the parameter distribution allows for a wide range of values, then the assumed quasiconcavity of the utility function comes into danger.

Example: A simple model for train ticket demand (continued). Burtless and Hausman [5] explicitly take into account the variation of tastes of different consumers. We also do this by adjusting the demand equation in (7) as follows:

$$d = kp^\alpha y^\beta.$$

In this equation the k 's represent individual variation in tastes. The authors model these as follows:

$$k = \exp(z'\delta + \zeta),$$

where z is a vector of individual characteristics and ζ is a stochastic error term. Technically it would not be a problem to let both the parameters α and β vary as well; the resulting parameters would be identified.

4.2 Estimation

In the large amount of literature that has appeared on nonlinear budget constraints there have been proposed different techniques for estimating the demand functions. The techniques that have been used up to present can be shared under one of the following estimation methods:

1. Ordinary least squares estimation (OLS)
2. Reduced form estimation (RF)
3. Instrumental variable estimation (IV)
4. Maximum likelihood estimation (ML)
5. Nonparametric estimation (NP)

We will give a short description of the first four methods. A description of the latter method is provided by Blomquist and Newey [4].

4.2.1 Ordinary least squares

Hall [8] raised the idea to linearize the budget constraints around observed demand and hence use the ordinary least squares (OLS) estimation procedure to generate estimates of the parameters. However, as Burtless and Hausman [5] note, this procedure generally yields biased estimates if a specific optimization / measurement error term is included. They give the following example:

Example: A simple model for train ticket demand (continued). Let $d(\cdot)$ be a demand function that is specified by

$$d = kp^\alpha y^\beta + \varepsilon,$$

where ε is a random error term that takes account of both optimization error and measurement error. Then it can be seen that (i) p and ε are correlated and that (ii) y and ε are correlated, whereas both variables are functions of the actual demand for train tickets.

This problem is often referred to as the *simultaneous equations problem*. In his investigation of small sample properties of various estimators Blomquist [3] concludes that the performance of OLS estimators in fact is very poor as would be expected. To overcome the endogeneity problem Burtless and Hausman [5] suggest the use of instrumental variable (IV) estimation.

4.2.2 Reduced form estimation

In the ‘reduced form’ approach the price variable is replaced by a linear system that determines the price. For instance in a model with two different segments this could be done as follows for the two good case:

$$d = \alpha_1 p_1 + \alpha_2 p_2 + \beta y + \gamma K + \varepsilon,$$

where K denotes the consumption level at which the marginal price changes from p_1 to p_2 . Herriges and King [10] note the following limitations of this estimation procedure:

- The model specification is rather *ad hoc*, providing little theoretical justification for which components are included in the model. Moreover, it is not derived from standard microeconomic utility maximizing behavior.
- As a result of this, it is often difficult to interpret the parameter estimates in a correct way.
- If the level K is constant throughout the sample then γ would not be identified. Hence it would not be possible to predict changes in consumption levels if K were changed.

4.2.3 Instrumental variable estimation

In the ‘instrumental variable’ estimation method one would form an instrument for the marginal price, p . For example,

$$p = \Pr\{p = p_1|z\} p_1 + \Pr\{p = p_2|z\} p_2,$$

where z denotes a vector of individual characteristics (like e.g. income) that influences consumers in choosing their optimal consumption levels. The probabilities $\Pr\{p = p_i|z\}$, $i = 1, 2$ should have a parametric specification that can be estimated by Ordinary Least Squares (OLS) or Maximum Likelihood (ML) techniques. The predicted values for p , say \hat{p} , should then be used in an OLS regression. The endogeneity problem is accounted for in this method, as price (in this case \hat{p}) does not depend on quantity anymore.

Herriges and King [10] mention that the IV procedure is technically superior to the RFE method, as the demand equations are specified within the neoclassical microeconomic framework. On the other hand, the IV method has got some drawbacks as well:

- The IV estimation separates the simultaneous discrete / continuous choice decision into two separate decisions. In the first stage of the IV procedure the consumer price is determined. This is the discrete choice concerning the segment on which a consumer wants to locate himself. In the second stage the exact consumer demand is determined, given his location on a specific segment. It is of course if the model takes account of the fact that the discrete / continuous choice decision is taken simultaneously by consumers.
- Estimating the elasticity of demand is often difficult as price and usage equations are separated.
- The IV estimation procedure is less efficient than the ML approach, that will be discussed next.

4.2.4 Maximum likelihood estimation

A good account of this estimation procedure can be found in Moffitt [11].

5 Modeling consumer demand for train tickets/train kilometers

According to Bhat and Pulugurta [1] consumer travel demand can be modeled in two different ways: the ordered response mechanism and the unordered response mechanism.

In the ordered response mechanism the discrete choice of a consumer corresponds to a certain interval of an underlying latent continuous variable. Despite the fact that many studies of travel demand have

made use of this mechanism it has one major drawback: It is at odds with the - widely adopted - *utility maximizing postulate*.

In the unordered response mechanism the choice determining structure of consumer demand is not restricted to be a single continuous variable. Instead, one can introduce a utility structure to model consumer preferences and derive consumer demand (or vice versa). In modeling travel demand different kinds of utility structures have been introduced. Many authors have made use of discrete choice models, the most prominent being the (nested) logit model. Discrete choice models combine a strong theoretical foundation with analytical convenience. This analytical convenience is achieved however at the cost of a functional structure that may be too restrictive (Oum and Gillen [13]). Moreover, by their nature discrete choice models focus on the discrete choice, whereas in many travel demand decisions consumers typically face a *simultaneous* discrete/continuous choice.

The approach to let a discrete and a continuous choice simultaneously arise from a single utility framework was more or less introduced by Burtless and Hausman [5] in the context of labor supply, and has been used since to model various consumer decisions where both a discrete and a continuous choice have to be made simultaneously. Amongst others one can find applications in the fields of water demand, electricity demand, disability insurance and the demand for housing. We will make use of a discrete/continuous choice model to model demand for train ticket tickets/train kms. The consumer's choice of train ticket is his discrete choice and his total consumption of train kms is his continuous choice. The train ticket choice determines the fixed price as well as the marginal price that one has to pay. The standard solution to this *endogeneity* problem is to rewrite the model and make use of the maximum likelihood procedure, as was outlined in section 4.

5.1 Discrete models

The *ordered probit* model for train ticket choice:

$$y = \alpha + \beta x + \varepsilon, \quad \varepsilon \sim N(0, 1)$$

$$t = \begin{cases} 1 & \text{if } y \leq \mu_1 \\ 2 & \text{if } \mu_1 < y \leq \mu_2 \\ 3 & \text{if } y \geq \mu_2 \end{cases}$$

Train ticket choice is denoted by t , y is a latent unobserved variable, and x is a vector with individual characteristics. Note that this model completely neglects the demand for train kms as it only focuses on the (discrete) train ticket choice. Instead it makes use of the artificial variable y . As mentioned before, another major drawback of this kind of ordered models is that there is no sound microeconomic foundation for them. In other words, the equations cannot be thought of as resulting from a (random) utility framework.

Another discrete choice model is the *multinomial logit* model for train ticket choice. This model can e.g. be found in Cramer [6]. A great advantage of this model over the ordered probit model is that it is consistent with (random) utility theory. The other drawback however, the neglect of the demand for train kms, also holds for this model.

5.2 Continuous models

A linear regression model for the logarithm of train kilometer demand k :

$$k = \alpha + \beta p_t + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2),$$

where p_t is the marginal cost of a traveled kilometer given the type of travel card t chosen by the individual. As was mentioned in section 4.2.1 OLS estimation of this equation yields biased estimates due to the endogeneity of the price p_t .

5.3 Simultaneous discrete/continuous models

We now turn to a more realistic model, which incorporates the simultaneity of train kms and train tickets. Throughout this subsection we will denote the logarithmic transformation of the demand for train kms by k .

$$k = \alpha + \gamma_2 t_2 + \gamma_3 t_3 + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2) \quad (9a)$$

$$t_i = \begin{cases} 1 & \text{if } k \in (\mu_{i-1}, \mu_i] \\ 0 & \text{otherwise} \end{cases} \quad (9b)$$

$$\mu_i = m_i + \eta, \quad \eta \sim N(0, \tau^2) \quad (9c)$$

where $-\infty = m_0 < m_1 < m_2 < m_3 = \infty$. Equation (9a) is the demand function for train kms, where demand depends on dummy variables relating to the type of card owned (t_1, t_2 or t_3). Equation (9b) describes how the choice of travel card depends on kilometers traveled, while eqrefstreq3 makes the stochastic component in this choice explicit. The stochastic term ε takes account of specification error, while the optimization error by the consumer is modeled by η (cf. section 4.1). The optimization error term plays a stronger role in this application than in other applications, such as labor supply and electricity demand. The reason is that consumers play a more active role than in other applications, as they *explicitly* choose both their consumption of train kms and their price (by choosing their train tickets). In most other applications a consumer chooses his optimal level of consumption, thereby *automatically* choosing his price level. Thus in these applications the discrete choice has a more *implicit nature* than in our case.

It is assumed that ε and η are uncorrelated. The likelihood function for this model is given by²

$$\begin{aligned} p(k, t_i | \theta) &= \Pr \{k = \alpha + \gamma_i t_i + \varepsilon; k \in (\mu_{i-1}, \mu_i]\} \\ &= \Pr \{\varepsilon = k - \alpha - \gamma_i t_i; \eta_{i-1} < k - m_{i-1}; \eta_i \geq k - m_i\} \\ &= \frac{1}{\sigma} \phi\left(\frac{k - \alpha - \gamma_i t_i}{\sigma}\right) \Phi\left(\frac{k - m_{i-1}}{\tau}\right) \left[1 - \Phi\left(\frac{k - m_i}{\tau}\right)\right], \end{aligned} \quad (10)$$

where $\gamma_1 \equiv 0$, $\theta := (\alpha \ \gamma_2 \ \gamma_3 \ m_1 \ m_2 \ \sigma \ \tau)$.

As Moffitt [11] has pointed out, there is much reason to believe that a term for *heterogeneity of preferences* should be added. This can be achieved through modifying the model in (9) by replacing the

²The following expressions should not be regarded as probabilities but rather as density functions.

constant α by $\alpha + \delta z$ where the vector z contains variables that affect consumers' taste, like e.g. 'car availability'.

Thus far the demand equation for trainkilometers only contained a constant and variables concerning train ticket characteristics. In order to construct a demand equation that is more in line with microeconomic theory, one could add *instruments for income*, like e.g. 'education'. Including such instruments in the vector w , the model now becomes

$$k = \alpha + \beta w + \gamma_2 t_2 + \gamma_3 t_3 + \delta z + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2) \quad (11)$$

$$t_i = \begin{cases} 1 & \text{if } k \in (\mu_{i-1}, \mu_i] \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$\mu_i = m_i + \eta, \quad \eta \sim N(0, \tau^2) \quad (13)$$

Instead of "train ticket type" it would be preferable - from an economic point of view - to include price characteristics into the demand equation in (9). One can think of two additional variables, i.e. "fixed price" and "marginal price". This is left as a topic for further research.

In the above models we have made the assumption that the stochastic terms for optimization error and heterogeneity of preferences are independent, i.e. $\mathbb{E}[\eta_i \eta_j] = \mathbb{E}[\eta_i] \mathbb{E}[\eta_j]$, $i \neq j$. It is likely however that this assumption will not hold in reality, as an upward bias of the one μ_i is likely to coincide with an upward bias of another μ_j . It is therefore possible to modify the distributional assumption of η to

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \sim N(0, T).$$

Note that it is well possible that there are consumers who travel with different types of train tickets during a month. In the models that have been discussed this far all respondents have to be categorized into one 'ticket group' however. Therefore a respondent is characterized as a 'ticket type i consumer' if he travels most kilometers (trips) with this type of ticket.

In most household demand models consumers choose their optimal consumption bundles, hence purchase these at some given price, and finally consume them. This consumption is mostly assumed to have zero cost. In case this latter assumption does not hold, an adjustment should be made to the budget constraint in (6).

To return to the case of consumption of trainkilometers/train tickets, it is clear that the *actual* consumption of a train kilometer induces a cost for the consumer. The cost of the consumption of a train kilometer is for the most part a matter of time valuation: Some people may value a minute spent in train very low, others find it useful to read a book or even work. Related issues are the valuation of waiting time spent on platforms, the valuation of time spent in pre- and after-transport modes and the valuation of uncertainty concerning punctuality (i.e. the valuation of a "risky minute"). These factors all have some influence on the *generalized price* of a train kilometer and should therefore - at least in some way - be incorporated into the model.

6 Data

In this paper we study a dataset from the Dutch Railway (NS), the so-called 'Basisonderzoek' (BO). This dataset was obtained through a survey during the period April 1992 until March 1993. The most important objective of the study was to analyze the profiles of travelers, in particular those of travelers by train. The survey consisted of two parts: At first a random sample from the population of Dutch households was drawn, after which every household was asked about personal characteristics like age, sex and education. Moreover the subjects were asked about their possession of train fare reduction tickets, distance to the nearest train station and other factors of interest for the Dutch Railway. The second part of the survey concentrated on households that traveled by train. Every subject was asked to keep a one month diary of his travel behavior as far as railway transport was concerned.

In fact the dataset consists of two parts: The first part is a cross section dataset of the Dutch population. The subjects in the second dataset are a subsample of those in the first, satisfying two selection criteria: (i) the subject indicated that he traveled by train during the past year, and (ii) the subject was willing to keep a travel diary. This latter criterion may be interesting, because it can lead to some form of selection bias if one is willing to estimate relations between certain variables. Hence it is seen that the second dataset is a repeated cross section dataset, as there is a time element - twelve months - but the household sample changes every period. The data set contains 84054 entries. It can be divided into three different parts:

- people who travel by train and have kept a travel diary (10911 entries),
- people who travel by train but did not keep a travel diary (29965 entries),
- people who do not travel by train (43178 entries).

In this paper we will consider the datasets as one big cross section dataset. Practically speaking we will in some cases add variables to the first dataset, such as the total number of trainkilometers and the number of trips. For other cases the data will remain the same, as not all households participated in the second part of the survey.

It should be noted that we will neglect some part of the data in the next section: This concerns those respondents who did not have to make a real choice, but instead got their train tickets for free. The most prominent are: students, people from the army and railway personnel.

7 Estimations

Estimates of the discrete/continuous choice model in (9) as well as estimates for the model in (11) can be found in Table 4. The data consisted of those people who had kept a travel diary (see section 6). The train km consumption was measured on a monthly basis.

According to the model consumers tend to buy a reduced fare pass if they consume at least $\exp(m_1) = 318.9$ train kms. For seasonal tickets the border value equals $\exp(m_2) = 757.5$ kms.

The expected consumption of train kms given that one does not possess a reduced fare ticket nor a seasonal ticket equals $\exp\left(\alpha + \frac{\sigma^2}{2}\right) = 228.4$ kms. The expected number of train kilometers for reduced fare travelers and owners of seasonal tickets is $\exp\left(\alpha + \gamma_2 + \frac{\sigma^2}{2}\right) = 483.1$ kms and $\exp\left(\alpha + \gamma_3 + \frac{\sigma^2}{2}\right) = 954.5$ kms respectively.

	average kms
full tariff	223.37
reduced tariff	417.03
seasonal tickets	818.87
all respondents	396.10

Table 3: Mean demands for given train tickets

	Model (9)		Model (11)	
Mean log-likelihood	-2.34555		-2.32153	
Parameters	Estimates	Standard error	Estimates	Standard error
α	4.824	0.015	5.046	0.033
γ_2	0.749	0.024	0.726	0.025
γ_3	1.430	0.029	1.439	0.030
m_1	5.765	0.031	5.765	0.031
m_2	6.630	0.039	6.629	0.039
σ	1.102	0.008	1.076	0.007
τ	2.092	0.034	2.091	0.034
25-	-	-	-0.105	0.030
60+	-	-	-0.009	0.033 ³
female	-	-	-0.212	0.022
car ++	-	-	-0.234	0.025
car +	-	-	-0.106	0.033
edu +	-	-	0.095	0.027
edu ++	-	-	0.303	0.027
unemployed	-	-	-0.232	0.027

Table 4: Estimation results

It is seen that estimates of the structural parameters of the model hardly change if the personal characteristics are incorporated into the model. The personal characteristics consist of age dummy variables “younger than 25”, “older than 60”, a dummy for female respondents and dummy variables for car availability and education. We have also included a dummy variable for unemployed people.

8 Conclusions and directions for further research

In this paper we have proposed a model that describes the consumer behavior for travelers by train. We have estimated a first version of our model, the results are found in Table 4. We realize that there are still many ways to improve our model, as was discussed in section 5.3, and also in other sections. This is left for the (near) future.

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